## Computer Science for Biology (algorithmic primer)

## Overview of Quiz 0

## Note: I grabbed these stats last night, so the distribution may have changed by this morning.

From among the following answers, select the one that best describes how well you believe you know the topic of asymptotic analysis.

| extremely well | 3 respondents | $\mathbf{6 \%}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| fairly well | 16 <br> respondents | $\mathbf{3 0 \%}$ |  |
| so-so | 19 <br> respondents | $\mathbf{3 6 \%}$ |  |
| I've heard of it before, but don't really | 4 respondents | $\mathbf{8 \%}$ |  |
| know it | 13 <br> respondents | $\mathbf{2 5 \%}$ |  |
| new semester, who dis? |  |  |  |

## Overview of Quiz 0

From among the following answers, select the one that best describes how well you believe you know the topic of dynamic programming.


## Overview of Quiz 0

From among the following answers, select the one that best describes how well you believe you know the topic of computational complexity.


## Overview of Quiz 0

```
Given a directed acyclic graph (DAG) G = (V,E), give a (tight) asymptotic
upper bound for the complexity of finding the shortest path between two
vertices s, t f V .
```

Key knowledge: Every DAG has a topological order - a way you can visit the nodes so that when you see a node, you've already seen every node that has an edge to it.

## Overview of Quiz 0

Given a directed acyclic graph (DAG) $G=(\mathrm{V}, \mathrm{E})$, give a (tight) asymptotic upper bound for the complexity of finding the shortest path between two vertices $s, t \in V$.

Key knowledge: Every DAG has a topological order - a way you can visit the nodes so that when you see a node, you've already seen every node that has an edge to it.

One can find a topological order for a DAG in linear $O(V+E)$ time.

Given the topological order, shortest path can easily be computed in $O(V+E)$ time.

## Overview of Quiz 0

I am given two coins, which I will flip simultaneously.
Let $X=H$ be the event that the first coin comes up heads and $X=T$ be the event that the first coin comes up tails. Let $Y=H$ and $Y=T$ be defined similarly for the second coin.

If you know that $\operatorname{Pr}(\mathrm{X}=\mathrm{H}, \mathrm{Y}=\mathrm{H})=0.25$ and $\operatorname{Pr}(\mathrm{X}=\mathrm{T}, \mathrm{Y}=\mathrm{H})=0.5$ and $\operatorname{Pr}(\mathrm{X}=\mathrm{H}, \mathrm{Y}=\mathrm{T})=0.05$.

What is $\operatorname{Pr}(X=T, Y=T)$ ?
C1\C2 H T

H
0.25
0.05

T 0.5 ?

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$\begin{array}{ll}\text { C1 } 1 \mathrm{C} 2 & \mathrm{H}\end{array}$
What is $\operatorname{Pr}(X=T, Y=T)$ ?
Prob of all possible events must sum to 1 (unitary)
0.25
0.05

T
0.5
0.2

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$$
\begin{array}{lll}
\text { C1\C2 } & H & \text { T }
\end{array}
$$

What is $\operatorname{Pr}(X=T)$ ?
H
0.25
0.05
$\begin{array}{lll}\mathrm{T} & 0.5 & 0.2\end{array}$

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$$
\begin{array}{lll}
\text { C1\C2 } & H & \text { T }
\end{array}
$$

What is $\operatorname{Pr}(X=T)$ ?

The marginal prob. is the sum over all joint states where

H
0.25
0.05
0.3 X has this value.

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Are the two coin flips statistically independent (say why or why not)?

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Are the two coin flips statistically independent (say why or why not)?

No: $X$ and $Y$ are statistically independent iff
$P(X=x, Y=y)=P(X=x)^{*} P(Y=y)$

## Overview of Quiz 0

The program grep allows one to quickly find a target pattern in a file. In practice grep works quickly by using a combination of a well-engineered implementation, and an efficient algorithm for pattern search. Assume that you are given a large string $T$ representing the file and a string $P$ representing the pattern the user wishes to find.

What algorithm might you use to search for the pattern P in T ?

What is the running time of your algorithm in terms of $|\mathrm{T}|$ and $|\mathrm{P}|$ (the lengths of strings $T$ and $P$, respectively)?

Many possible answers, but an efficient algorithm will be linear in the length of the input $\mathrm{O}(|\mathrm{T}|+|\mathrm{P}|)$, we will learn about such algorithms next week.

## What is Computer Science?



## What is Computer Science?

Not actually simple to define constructively

Still debate whether certain areas constitute CS

Computer science is the scientific and practical approach to computation and its applications. It is the systematic study of the feasibility, structure, expression, and mechanization of the methodical procedures (or algorithms) that underlie the acquisition, representation, processing, storage, communication of, and access to information* ...

## What is Computer Science?

Concerned with the development of provably correct and efficient computational procedures (algorithms \& data structures) to answer well-specified problems.

To answer a computational question, we first need a well-formulated problem.

It turns out that a major challenge in bioinformatics will simply be determining how to frame the computational problem corresponding to a biological question in a well-posed and meaningful way!

## Assembly

Reads


Referenco genomo


How to assemble puzzle without the benefit of knowing what the finished product looks like?

## Assembly

Whole-genome"shotgun" sequencing starts by copying and fragmenting the DNA
("Shotgun" refers to the random fragmentation of the whole genome; like it was fired from a shotgun)

$$
\begin{aligned}
\text { Input: } & \text { GGCGTCTATATCTCGGCTCTAGGCCCTCATTTTTT } \\
\text { Copy: } & \text { GGCGTCTATATCTCGGCTCTAGGCCCTCATTTTTT } \\
& \text { GGCGTCTATATCTCGGCTCTAGGCCCTCATTTTTT } \\
& \text { GGCGTCTATATCTCGGCTCTAGGCCCTCATTTTTT } \\
& \text { GGCGTCTATATCTCGGCTCTAGGCCCTCATTTTTT }
\end{aligned}
$$

Fragment: GGCGTCTA TATCTCGG CTCTAGGCCCTC ATTTTTT GGC GTCTATAT CTCGGCTCTAGGCCCTCA TTTTTT GGCGTC TATATCT CGGCTCTAGGCCCT CATTTTTT GGCGTCTAT ATCTCGGCTCTAG GCCCTCA TTTTTT

## Assembly

Assume sequencing produces such a large \# fragments that almost all genome positions are covered by many fragments...


## Assembly

...but we don't know what came from where

|  | CTAGGCCCTCAATTTTT |
| :--- | :--- |
|  | GGCGTCTATATCT |
| Reconstruct |  |
| this | CTCTAGGCCCTCAATTTTT |
|  | TCTATATCTCGGCTCTAGG |
|  | GGCTCTAGGCCCTCATTTTTT |
|  | CTCGGCTCTAGCCCCTCATTTT |
|  | TATCTCGACTCTAGGCCCTCA |
|  | GGCGTCGATATCT |
|  | TATCTCGACTCTAGGCC |
|  | GGCGTCTATATCTCG |
|  | GGCGTCTATATCTCGGCTCTAGGCCCTCATTTTTT |

## Assembly

|  | CTAGGCCCTCAATTTTT |
| :--- | :--- |
|  | GGCGTCTATATCT |
|  | CTCTAGGCCCTCAATTTTT |
| Reconstruct this | TCTATATCTCGGCTCTAGG |
|  | GGCTCTAGGCCCTCATTTTT |
|  | CTCGGCTCTAGCCCCTCATTTT |
|  | TATCTCGACTCTAGGCCCTCA |
|  | GGCGTCGATATCT |
|  | TATCTCGACTCTAGGCC |
| GGCGTCTATATCTCG |  |
| $\longrightarrow ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ~$ |  |

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Concerned with the development of provably correct and efficient computational procedures (algorithms \& data structures) to answer well-specified problems.

To answer a computational question, we first need a well-formulated problem.

Given: a collection, R, of sequencing reads (strings)

Find: The genome (string), G, that generated them

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Given: a collection R, of sequencing reads (strings)
Find: Theserlome (string), G, that gerrereted them
Not well-specified.
What makes one genome more likely than another?
What constraints do we place on the space of solutions?

## What is Computer Science?

Concerned with the development of provably correct and efficient computational procedures (algorithms \& data structures) to answer well-specified problems.

To answer a computational question, we first need a well-formulated problem.

Given: a collection, R, of sequencing reads (strings)
Find: The shortest genome (string), G, that contains all of them

## Shortest Common Superstring

Given: a collection, $S=\left\{s_{1}, s_{2}, \ldots, s_{k}\right\}$, of sequencing reads (strings)

Find*: The shortest possible genome (string), G, such that $s_{1}, s_{2}, \ldots, s_{k}$ are all substrings of G

How, might we go about solving this problem?
*for reasons we'll explore later, this isn't actually a great formulation for genome assembly.

## Shortest common superstring

Given a collection of strings $S$, find $\operatorname{SCS}(S)$ : the shortest string that contains all strings in $S$ as substrings

Without requirement of "shortest," it's easy: just concatenate them
Example: S: BAA AAB BBA ABA ABB BBB AAA BAB
Concatenation: BAAAABBBAABAABBBBBAAABAB


SCS(S): AAABBBABAA


AAA
AAB
ABB
BBB
BBA
BAB
ABA
BAA

Idea: pick order for strings in S and construct superstring

## order 1: $\underset{\downarrow}{\text { AAA AAB }}$ ABA ABB BAA BAB BBA BBB <br> AAA

Idea: pick order for strings in S and construct superstring

## order 1: $\underset{\downarrow}{\text { AAA AAB }}$ ABA ABB BAA BAB BBA BBB <br> AAAB

Idea: pick order for strings in S and construct superstring

order 1: AAA AAB ABA $A B B$ BAA BAB BBA BBB AAABA

Idea: pick order for strings in S and construct superstring

## order 1: AAA AAB $\underset{\text { ABA } A B B}{ } B A A$ BAB BBA BBB AAABABB

Idea: pick order for strings in S and construct superstring

## order 1: AAA AAB ABA ABB BAA BAB BBA BBB AAABABBAABABBABBB $\longleftarrow$ superstring 1

Idea: pick order for strings in S and construct superstring
order 1: AAA AAB ABA ABB BAA BAB BBA BBB AAABABBAABABBABBB $\longleftarrow$ superstring 1
order 2: AAA AAB ABA BAB ABB BBB BAA BBA AAABABBBAABBA $\longleftarrow$ superstring 2

Try all possible orderings and pick shortest superstring
If $S$ contains $n$ strings, $n!$ ( $n$ factorial) orderings possible

## order 1: AAA AAB ABA ABB BAA BAB BBA BBB AAABABBAABABBABBB « superstring 1

order 2: AAA AAB ABA BAB ABB BBB BAA BBA AAABABBBAABBA $\longleftarrow$ superstring 2

If $S$ contains $n$ strings, $n!$ ( $n$ factorial) orderings possible

## Shortest common superstring

Can we solve it?
Imagine a modified overlap graph where each edge has cost $=-$ (length of overlap)

SCS corresponds to a path that visits every node once, minimizing total cost along path

That's the Traveling Salesman
Problem (TSP), which is NP-hard!
$S$ : AAA AAB ABB BBB BBA
SCS(S): AAABBBA
AAA
AAB


## Shortest common superstring

Say we disregard edge weights and just look for a path that visits all the nodes exactly once

That's the Hamiltonian Path problem: NP-complete

Indeed, it's well established that SCS is NP -hard

S: AAA AAB ABB BBB BBA
SCS(S): AAABBBA
AAA AAB
ABB


## Shortest common superstring \& friends

Traveling Salesman, Hamiltonian Path, and Shortest Common Superstring are all NP-hard

For refreshers on Traveling Salesman, Hamiltonian Path, NP-hardness and NP-completeness, see Chapters 34 and 35 of "Introduction to Algorithms" by Cormen, Leiserson, Rivest and Stein, or Chapters 8 and 9 of "Algorithms" by Dasgupta, Papadimitriou and Vazirani (free online: http://www.cs.berkeley.edu/~vazirani/algorithms)

> Important note: The fact that we modeled SCS as NPhard problems (TSP and HP) does not prove that (the decision version of) SCS is NP-complete. To do that, we must reduce a known NP-complete problem to SCS.

Given an instance I of a known hard problem, generate an instance l' of SCS such that if we can solve l' in polynomial time, then we can solve I in polynomial time. This implies that SCS is at least as hard as the hard problem.

This can be done e.g. with HAMILTONIAN PATH


## Shortest Common Superstring

The fact that SCS is NP-complete means that it is unlikely that there exists any algorithm that can solve a general instance of this problem in time polynomial in n - the number of strings.

If we give up on finding the shortest possible superstring $G$, how does the situation change?

## Shortest Common Superstring

There's a "greedy" heuristic that turns out to be an approximation algorithm (provides a solution within a constant factor of the the optimum)

## Different approx. (not all greedy)

At each step, chose the pair of strings with the maximum overlap, merge them, and return the merged string to the collection.

Greedy conjecture factor of 2 OPT is the worst case

| ratio | authors | year |
| :--- | :--- | ---: |
|  | approximating SCS |  |
| 3 | Blum, Jiang, Li, Tromp and Yannakakis [4] | 1991 |
| $2 \frac{8}{9}$ | Teng, Yao [23] | 1993 |
| $2 \frac{5}{6}$ | Czumaj, Gasieniec, Piotrow, Rytter [8] | 1994 |
| $2 \frac{50}{63}$ | Kosaraju, Park, Stein [15] | 1994 |
| $2 \frac{3}{4}$ | Armen, Stein [1] | 1994 |
| $2 \frac{50}{69}$ | Armen, Stein [2] | 1995 |
| $2 \frac{2}{3}$ | Armen, Stein [3] | 1996 |
| $2 \frac{25}{42}$ | Breslauer, Jiang, Jiang [5] | 1997 |
| $2 \frac{1}{2}$ | Sweedyk [21] | 1999 |
| $2 \frac{1}{2}$ | Kaplan, Lewenstein, Shafrir, Sviridenko [12] 2005 |  |
| $2 \frac{1}{2}$ | Paluch, Elbassioni, van Zuylen [18] | 2012 |
| $2 \frac{11}{23}$ | Mucha [16] | 2013 |

## Greedy shortest common superstring



## Greedy shortest common superstring



## Greedy shortest common superstring



## Greedy shortest common superstring



## Greedy shortest common superstring



## Greedy shortest common superstring



## Greedy shortest common superstring



## Greedy shortest common superstring



## Greedy shortest common superstring

## AAABBBA $\longleftarrow$ superstring, length=7

## Shortest common superstring: greedy

Greedy-SCS: in each round, merge pair of strings with maximal overlap. Stop when there's 1 string left. $l=$ minimum overlap.

Algorithm in action ( $l=1$ ):
$\longmapsto$ Input strings ——
AAA AAB ABB BBB BBA


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AAA AAB ABB BBB BBA
AAAB ABB BBB BBA
AAAB BBBA ABB
AAABB BBBA


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Algorithm in action ( $l=1$ ):
$\longmapsto$ Input strings ——
AAA AAB ABB BBB BBA
AAA AAB ABB BBB BBA
AAAB ABB BBB BBA
AAAB BBBA ABB

AAABB BBBA
AAABBBA

## AAABBBA

That's the SCS

# Greedy shortest common superstring 

AAA AAB ABB BBA BBB AAAB ABB BBA BBB

# Greedy shortest common superstring 

AAA AAB ABB BBA BBB<br>AAAB ABB BBA BBB<br>AAAB ABBA BBB

# Greedy shortest common superstring 



## Greedy shortest common superstring



## Greedy shortest common superstring

> Note: approx. guarantee is on length of the superstring Actual result may be very different.


AAABBBA $\longleftarrow$ superstring, length=7

Greedy answer isn't necessarily optimal

## Take-home message:

We are interested in correct and efficient algorithms for solving well-specified problems.

We must be careful about how we pose the problems.

Actually, shortest common superstring is a rather poor model for sequence assembly, due to repeats and errors.

